CHAMP Attitude Aberration Correction

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1 Scope

The CHAMP Advanced Stellar Compass ASC is capable of performing in-flight aberration correction. Full functionality became operational in February 2001. Before that, during the CHAMP instrument commissioning phase, the autonomous aberration correction was not enabled. Post-processing ASC data from the first months of the mission thus requires to consider the aberration effect.

In chapter 3 an overview of the effect is given. The annual and orbital aberration are explained and basic formulas are given.

In chapter 4 the mathematical framework and a possible method to compute and apply the corrections is given.

Finally in chapter 5 the initial results from in-flight aberration correction performed after full functionality, as have been reached, are presented.

2 Reference Documents

[RD03] "The Astronomical Almanac 2001"

3 Aberration Correction

CHAMP carries 2 ASCs, each operating 2 camera head units (CHU). Two CHUs are part of the boom-mounted magnetometry assembly, while the other two are mounted to the accelerometer, housed by the satellite body. The boresight of each CHU is orthogonal on the CCD-chip and roughly perpendicular to the satellite velocity vector in nominal attitude with the boom pointing in flight direction.
For a body (i.e. observed star) in the solar system the apparent direction at the instant of observation differs from the geometric direction at that instant because of the relative motion of the observer (here ASC) and the body. This light-time effect becomes significant in case high precision observations are performed at high relative motion.

For an Earth-orbiting spacecraft, the motion of the Earth around the sun causes the annual aberration, while the motion of the spacecraft about the Earth accounts for an additional orbital aberration effect. The resulting velocity of the observer needs to be considered in the same reference frame as the star observations are performed, i.e. in a quasi-inertial reference frame.

\[ \sin \gamma = \frac{c \sin \phi}{v} = \frac{c}{c} \]  
\[ (\text{Eq. 3.1}) \]

using \( \sin \gamma \equiv \gamma \) (for \( \gamma \ll 1 \)) the aberration angle \( \gamma \) may be computed from the observer (spacecraft) velocity by:
with \( c = 2.997925 \times 10^5 \text{ km/s} \) being the speed of light, \( v \) the velocity of the observer in km/s, and thus

\[
\gamma = 0.688025 \cdot v \cdot \sin \phi \quad [\text{arc sec}]
\]  
(Eq. 3.3)

Note that the aberration effect is most significant for stars perpendicular to the velocity vector (\( \phi = 90^\circ \)).

The annual aberration, caused by the velocity of the Earth, may be derived from common astronomical algorithms. The orbital aberration needs to be calculated from position and velocity of the spacecraft. The latter information is available from e.g. an on-board GPS receiver in the ITRF frame (see below), thus a transformation into the ICRF frame needs to be applied.

### 3.1 Annual Aberration

In this chapter the light-time effect due to the motion of the Earth around the Sun is described. The effect is called annual aberration.

The following formulas [RD04] are applicable to calculate the position of the sun (accuracy \( \sim 0.01 \)), derive the Earth velocity and eventually the annual aberration correction.

Due to the minor eccentricity of the Earth orbit, the annual aberration may be considered constant, introducing an error of \( \sim 0.35^\circ \) as described in section 3.1.1.

\( T_U \) is the interval of time, measured in Julian centuries elapsed since epoch 2000 January 1, \( 12^h \) UT. It is needed to calculate the mean anomaly, \( M \), of the sun:
The eccentricity $e$ of the Earth orbit around the sun is:

$$e = 0.016708617 - 0.000042037 \cdot T_U - 0.0000001236 \cdot T_U^2$$  \hspace{1cm} (Eq. 3.6)

In order to derive the true anomaly $\nu$ one needs to solve Kepler’s equation and find the eccentric anomaly $E$:

$$E = M + e \sin E \quad \text{and thus} \quad \tan \frac{\nu}{2} = \frac{1 + e}{1 - e} \tan \frac{E}{2}$$  \hspace{1cm} (Eq. 3.7)

Alternatively for orbits with small eccentricity $e$, a series expansion can be used to calculate $C$, being the difference $\nu - M$:

$$C = +(1^\circ \cdot 9146 - 0^\circ \cdot 004817 \cdot T_U - 0^\circ \cdot 000014 \cdot T_U^2) \cdot \sin M + (0^\circ \cdot 019993 - 0^\circ \cdot 000101 \cdot T_U) \cdot \sin 2M + 0^\circ \cdot 000029 \sin 3M$$  \hspace{1cm} (Eq. 3.8)

thus the true anomaly $\nu$ of the sun becomes

$$\nu = M + C$$  \hspace{1cm} (Eq. 3.9)

the radius vector $R$ of the sun given in astronomical units (1 AU = $1.4959787 \cdot 10^{11}$m)

$$R = \frac{1.000001018 \cdot \left(1 - e^2\right)}{1 + e \cos \nu}$$  \hspace{1cm} (Eq. 3.10)

and finally the velocity $v$ of the Earth at a given radius vector $R$:

$$v^2 = \mu_s \left(2 \frac{1}{R} - \frac{1}{a}\right)$$  \hspace{1cm} (Eq. 3.11)
with the sun semimajor axis $a = 1.4959787 \times 10^{11}$ m and the heliocentric gravitational constant $\mu_S = 1.32712438 \times 10^{20}$ m³/s².

With equation (Eq.3.2) we can than calculate the aberration angle $\gamma$ caused by the motion of the Earth:

$$\gamma = \frac{\mu_S}{c} \left( \frac{2}{R} - \frac{1}{a} \right) \sin \phi \quad [\text{rad}] \quad (\text{Eq. 3.12})$$

### 3.1.1 The Error when using a constant Earth velocity

An error for the annual aberration correction is introduced if a constant Earth velocity is assumed instead of considering e.g. a Keplerian orbit and considering the velocity at epoch. The Keplerian orbit of the Earth is an ellipse with semimajor axis, $a$, and eccentricity, $e$:

![Keplerian Orbit of the Earth](image.png)

Thus the minimum and maximum velocities of the Earth around the sun are at aphelion and perihelion, respectively:

$$v_{\text{min}} = \sqrt{\frac{\mu_S}{(1+e) \cdot a}} \left( \frac{2}{a} - \frac{1}{a} \right) \quad \text{and} \quad v_{\text{max}} = \sqrt{\frac{\mu_S}{(1-e) \cdot a}} \left( \frac{2}{a} - \frac{1}{a} \right) \quad (\text{Eq. 3.13})$$

resulting in

$$v_{\text{min}} = 29.29109 \text{ km/s}$$
$$v_{\text{max}} = 30.2865 \text{ km/s}$$
and the mean velocity $v_{\text{mean}}$ over one year

$$v_{\text{mean}} = 29.7822 \text{ km/s}$$

Using equation (Eq. 3.2) we can calculate the aberration effect for $\gamma_{\text{min}}$ and $\gamma_{\text{max}}$ for stars perpendicular to the velocity vector ($\phi=90^\circ$):

$$\gamma_{\text{min}} = 20.115''$$
$$\gamma_{\text{max}} = 20.837''$$
$$\gamma_{\text{mean}} = 20.491''$$

and thus obtain an annual aberration correction error of ~0.35" if a constant velocity of the Earth is used instead of the true velocity at epoch.

In the Astronomical Almanac [RD03] we find the constant of annual aberration for standard epoch 2000:

$$\gamma = 20.49552''$$

which shows that using a Keplerian orbit for the Earth is adequate to estimate the error effect as described.

Note that apart from the size of the velocity of the Earth, strictly also the direction needs to be taken into account. The change in direction will add a little (~0.1") to the total error due to the small eccentricity but has not been considered here.

### 3.2 Orbital Aberration

In this chapter the light-time effect due to the motion of the observer around the Earth is described. The effect is called orbital aberration.

On-board CHAMP the position and velocity are derived from GPS navigation solution, that are forwarded to the ASCs. This information is given in a different coordinate frame than that for processing the star images. The two coordinate frames and the transformation from one to the other are described in this section.

#### 3.2.1 Coordinate Systems

The GPS navigation solution contains the satellite position and velocity in the International Terrestrial Reference Frame (ITRF) :
$X_{\text{ITRF}}$ pointing towards the Greenwich meridian
$Y_{\text{ITRF}}$ pointing towards 90° east longitude
$Z_{\text{ITRF}}$ pointing to the north pole

The ASC star cameras provide and process the attitude in the International Celestial Reference Frame (ICRF):

- $X_{\text{ICRF}}$ pointing towards the vernal equinox
- $Y_{\text{ICRF}}$ pointing towards the summer point
- $Z_{\text{ICRF}}$ pointing to the north pole

The position of the Earth to the Sun with respect to the vernal equinox is depicted in Figure 3-5 below.

To transform a position from the ITRF into the ICRF frame not considering precession, nutation, and polar motion, the following rotation matrix applies:

$$
\begin{pmatrix}
\cos \Theta & -\sin \Theta & 0 \\
\sin \Theta & \cos \Theta & 0 \\
0 & 0 & 1
\end{pmatrix}
$$

(Eq. 3.14)

with $\Theta$ = Greenwich Hour Angle (Greenwich mean sidereal time). As depicted in Figure 3-5 below, the angle reflects the position of the Earth to the Sun with respect to the vernal equinox.
The Greenwich Hour Angle $\Theta$ is

$$\Theta = \Theta_0 + \Delta t \cdot \frac{d\Theta}{dt}$$  \hspace{1cm} (Eq. 3.15)

where $\Theta_0$ is the mean sidereal time at 0 h UT (cf. [RD03]) in seconds

$$\Theta_0 = 24110.54841 + 8640184.812866 \cdot T_U + 0.093104 \cdot T_U^2 - 6.2 \times 10^{-6} \cdot T_U^3$$  \hspace{1cm} (Eq. 3.16)

and $T_U$ is the interval of time as described in equation (Eq. 3.4).

The time derivative of the Greenwich Hour Angle reflects the rotation of the Earth in the ICRF during one solar day:

$$\frac{d\Theta}{dt} = \dot{\Theta} = 1 + 365.24219 \text{ revolutions per year}$$

$$= 15.0410682 \text{ deg/hour}$$

$$= 4.1780745 \times 10^{-3} \text{ deg/second}$$

Note that here $\Theta$ is calculated by considering the mean tropical year +1 revolution. More accurately a year is measured from vernal equinox to vernal equinox, and thus includes the average precession (46°/y). This effect is small as long as we are close to the origin of J2000.0.
3.2.2 Transforming the position and velocity vector from ECEF to IRF

The GPS navigation solution contains the position and velocity of the receiving object, i.e. the spacecraft CoM in the ITRF frame. From this data the orbital position $x$ and velocity $v$ in the ICRF frame can be derived with the approximations of eq. 3.14 by the following calculations:

\[ \mathbf{x}_{\text{IRF}} = (\mathbf{x}_{\text{RF,ECEF}})^{\text{RF,ECEF}} \cdot \mathbf{x}_{\text{ECEF}} \]  
\[ \text{(Eq. 3.17)} \]

\[ \mathbf{v}_{\text{IRF}} = (\mathbf{v}_{\text{RF,ECEF}})^{\text{RF,ECEF}} \cdot \mathbf{v}_{\text{ECEF}} + \frac{d(\mathbf{x}_{\text{RF,ECEF}})}{dt} \cdot \mathbf{x}_{\text{ECEF}} \]  
\[ \text{(Eq. 3.18)} \]

with

\[ (\mathbf{RF,ECEF})^{\text{RF,ECEF}} = \begin{pmatrix} \cos \Theta & -\sin \Theta & 0 \\ \sin \Theta & \cos \Theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \]  
\[ \text{(Eq. 3.19)} \]

\[ \frac{d(\mathbf{x}_{\text{RF,ECEF}})}{dt} = \begin{pmatrix} -\dot{\Theta} \cdot \sin \Theta & -\dot{\Theta} \cdot \cos \Theta & 0 \\ \dot{\Theta} \cdot \cos \Theta & -\dot{\Theta} \cdot \sin \Theta & 0 \\ 0 & 0 & 0 \end{pmatrix} \]  
\[ \text{(Eq. 3.20)} \]
4 APPLYING THE ABERRATION CORRECTION TO ASC DATA (ON-GROUND POSTPROCESSING)

During the early part of the CHAMP mission the aberration was not performed in orbit by the ASC data processing unit. Therefore it is necessary to include this step in the post processing software of the spacecraft attitude reconstruction. In this section we will outline the mathematical framework and computational steps required. All equations and constants will be given to the precision required for a proper correction of the derived attitude readings.

4.1 Calculation of Spacecraft Velocity

One important quantity is the total velocity of the observer. In our case it is the spacecraft velocity in the inertial reference frame (IRF):

\[ \vec{v}_{S/C} = \vec{v}_{ORB} + \vec{v}_{ANN} \]  

(Eq. 4.1)

where \( \vec{v}_{ORB} \) is the orbital velocity and \( \vec{v}_{ANN} \) the annual (Earth orbiting the sun).

In section 3 a detailed description of these velocity components has already been given. Here we take advantage of those formulae but restructure them slightly to make them fit this special application.

The basic equation for calculating the orbital velocity is

\[
\begin{pmatrix}
\cos \Theta & -\sin \Theta & 0 \\
\sin \Theta & \cos \Theta & 0 \\
0 & 0 & 1 \\
\end{pmatrix}
\begin{pmatrix}
\dot{\Theta} \cdot \cos \Theta & -\dot{\Theta} \cdot \sin \Theta & 0 \\
\dot{\Theta} \cdot \sin \Theta & \dot{\Theta} \cdot \cos \Theta & 0 \\
0 & 0 & 0 \\
\end{pmatrix}
\begin{pmatrix}
\vec{x}_{GPS} \\
\vec{v}_{GPS} \\
\end{pmatrix}
\]

(Eq. 4.2)

where \( \vec{x}_{GPS} \) and \( \vec{v}_{GPS} \) are the orbit position and velocity as derived from the GPS receiver, respectively. According to equation (Eq. 3.14) the Greenwich hour angle is

\[ \Theta = \Theta_0 + \dot{\Theta} \cdot (t - t_0) \]  

(Eq. 4.3)

where \( \dot{\Theta} = 7.29212 \cdot 10^{-5} \text{ rad/sec} \) reflects the angular velocity of the Earth rotation. For convenience we convert the sidereal angle into radians

\[ \Theta_0 = 1.75337 + 1.9910t \cdot 10^{-7} \cdot t_E \]  

(Eq. 4.4)

where \( t_E \) is the time elapsed since 1 Jan. 2000, 12 UTC in seconds.

\[ t_E = t_{GPS} - 630763213 \]  

(Eq. 4.5)
where $t_{\text{GPS}}$ is the actual GPS time in seconds. In this notation the sidereal angle, $\Theta_0$, is equivalent to $\pi$ in spring, $3\pi/2$ in summer, $2\pi$ in autumn and $\pi/2$ in winter. For equation (Eq. 4.3) we thus can write

$$\Theta = \Theta_0 + 7.27221 \cdot 10^{-5} \cdot (t_{\text{GPS}} - t_0)$$

(Eq. 4.6)

where $t_0$ is the time (GPS second) at the start of the actual day (00:00 UTC).

The annual velocity is the predominant part of the correction and it considers the motion of the Earth around the Sun. As pointed out in section 3 the Earth’s orbit is not circular. For the precision required here we may use a constant speed of $v = 29.782 \text{ km/sec}$ (max. error $\pm 1.66\%$). Taking into account the obliquity of the ecliptic plane ($\beta = 23^\circ.439096$ for epoch 2001, cf [RD03]) we obtain

$$\beta \Theta = \sin \cos v$$

(Eq. 4.7)

where $\Theta_0$ is the sidereal angle as defined in (Eq. 4.4).

4.2 Calculation of the Correction Angle

The apparent aberration of the light from a stellar object depends on the angle between the velocity of the observer and the direction to the object. Mathematically the deflection angle $\delta$ can be expressed as

$$\sin \delta = \frac{|\mathbf{v}_{\text{S/C}} \times \mathbf{z}_{\text{BORE}}|}{c}$$

(Eq. 4.8)

where $c = 299792.5 \text{ km/s}$ is the speed of light and $\mathbf{z}_{\text{BORE}}$ is a unit vector aligned with the boresight of the camera head unit (CHU). The direction of the boresight can directly be retrieved from the attitude reading of the star tracker.

There are two ways allowing to determine the boresight vector. One makes use of the attitude angles in the CHU frame. It is known that the direction of boresight can be expressed as

$$\mathbf{z}_{\text{BORE}} = \begin{bmatrix} \cos(\text{Decl}) \cdot \cos(\text{Right Ascension}) \\ \cos(\text{Decl}) \cdot \sin(\text{Right Ascension}) \\ \sin(\text{Decl}) \end{bmatrix}$$

(Eq. 4.9)
where Ra and Dec are the right ascension and the declination in the inertial reference frame. These angles can directly be derived from the attitude quaternion, $q_1$ through $q_4$.

$$\begin{align*}
Ra &= \text{ATAN2}\left((q_2q_3 - q_1q_4), (q_1q_3 + q_2q_4)\right) \\
\sin(Dec) &= -q_1^2 - q_2^2 + q_3^2 + q_4^2 \\
\cos(Dec) &= \sqrt{1 - \sin^2(Dec)}
\end{align*}$$

A more effective way is making use of the direction cosine matrix, DC, representing the attitude. The bottom row elements of this matrix directly reflect the vector components of the boresight.

$$\tilde{z}_{BORE} = \begin{pmatrix}
DC_{(3,1)} \\
DC_{(3,2)} \\
DC_{(3,3)}
\end{pmatrix} \quad \text{(Eq. 4.10)}$$

where $DC_{(3,i)}$ are the elements of the attitude direction cosine matrix.

The correction vector $\tilde{s}'$ in the inertial reference frame results from the cross product of the spacecraft velocity and the boresight.

$$\tilde{s}' = \tilde{v}_{S/C} \times \tilde{z}_{BORE} \quad \text{(Eq. 4.11)}$$

Before applying the correction to the initial attitude we have to transform $\tilde{s}'$ into the camera head frame with the help of the known direction cosine matrix.

$$\tilde{s} = DC \cdot \tilde{s}'$$

The correction vector, $\tilde{s}$, can conveniently be expressed in form of a quaternion.

$$\begin{pmatrix}
\frac{dq_1}{dt} \\
\frac{dq_2}{dt} \\
\frac{dq_3}{dt} \\
\frac{dq_4}{dt}
\end{pmatrix} = \frac{1}{2 \cdot c}
\begin{pmatrix}
s_X \\
s_Y \\
s_Z
\end{pmatrix}$$

$$dq_4 = \sqrt{1 - dq_1^2 - dq_2^2 - dq_3^2} \quad \text{(Eq. 4.12)}$$

where $c$ is the speed of light. For further processing we transform the quaternion into the matrix $M_{CORR}$. 

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Finally, the corrected attitude direction cosine matrix is obtained by multiplying the initial one with the transposed correction matrix:

\[ \text{DC}^\prime = M_{\text{CORR}}^T \cdot \text{DC} \quad \text{(Eq. 4.13)} \]

4.3 Applying the Correction

The CHAMP satellite is equipped with four star tracker camera head units (CHU). For convenience of the Attitude and Orbit Control System (AOCS) the attitude readings of all four CHUs are output in the same spacecraft fixed frame. The AOCS can thus switch from one CHU to the other, e.g. in case of invalid results, for adaption.

The first step in on-ground data processing of the ASC readings is therefore the transformation back into the CHU system. Subsequently the aberration correction should be applied. Particularly, in cases where the data of two adjacent CHUs are combined to obtain improved attitude readings in the Common Reference frame (CR) the aberration correction cannot be applied to the results afterwards.

A possible sequence of processing steps is:
5 RESULTS FROM IN-FLIGHT ABERRATION CORRECTION

With the latest CHAMP star camera software update the instrument is now able to perform in-flight the astronomical aberration correction.

The relativistic effect stems from the movement of the observer (star camera on spacecraft) to the apparent position of a star on the firmament. The velocity of the Earth is ~29.8 km/sec which needs to be considered to correct for the annual aberration, and by knowing the satellite velocity (CHAMP: ~7.6 km/sec) from GPS information it is possible to correct for the orbital aberration.

5.1 Without aberration correction

Figure 5-1 depicts the inter-boresight angle IBA between the two boom-mounted camera heads versus the satellite latitude (declination). In figure 5-2 the IBA as time series is given. The missing aberration correction causes an apparent deflection of the camera heads with respect to each other of up to ±40 arcsec. As a consequence the IBA varies elliptically over an orbit.

![IBA of camera heads vs. satellite latitude (aberration not corrected)](image1.png)

Figure 5-1  IBA of camera heads vs. satellite latitude (aberration not corrected)
Figure 5-2 IBA of camera heads as time series (aberration not corrected)

5.2 With aberration correction

The same kind of plots as above but for a time period where the in-flight aberration correction algorithm was enabled. Now the IBA is constant over an orbit. The flatness of the IBA versus declination demonstrates that there is no deformation of star tracker assembly due to thermal stresses from sun light and eclipses.

Figure 5-3 are superpositions of 4 consecutive orbits. The reproducibility is remarkable. Features re-appear at the same position along the orbit. The raw data scatter within a band of only 15 arcsec peak-peak. Postprocessing – merging the readings of both heads – will further reduce the noise to a few arcsec.
ASC-A Inter-Boresight-Angle

Feb. 17 2001, DOY 48

- Annual and orbital aberration correction enabled
- Peak-to-peak scatter ~15 arcsec
- Missing data due to sun blinding of one camera head

**Figure 5-3** IBA of camera heads vs. satellite latitude (aberration corrected)

ASC-A Inter-Boresight-Angle
(aberration correction enabled)
Feb. 17 2001, DOY 48

**Figure 5-4** IBA of camera heads as time series (aberration corrected)